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Optical gap solitons via second-harmonic generation: Exact solitary solutions

Claudio Conti,[1,2] Stefano Trillo,[1] and Gaetano Assanto[2]
[1]*Fondazione Ugo Bordoni, Via Baldassarre Castiglione 59, 00142 Roma, Italy*
[2]*Dipartimento di Ingegneria Elettronica, Terza University of Rome, Via della Vasca Navale 84, 00146 Roma, Italy*

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We report exact stationary localized solutions of the coupled-mode equations which describe two-color trapping through the interplay of (either single or double) Bragg coupling and second-harmonic generation. We check numerically their stability against small perturbations.
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Resonant three-wave interactions ($\omega_3=\omega_1+\omega_2$) or second-harmonic generation (SHG, $\omega_1=\omega_2$) as a degenerate case are widespread in physics. In homogeneous media soliton propagation in the presence of first-order dispersion was investigated nearly two decades ago [1,2]. Conversely, the possibility of sustaining solitary waves through the interplay of second-order dispersion (or diffraction) and parametric conversion has motivated recent efforts in optics [3–11]. Parametric solitons spatially confined in one [4], or two [5] transverse dimensions have been observed in uniform media.

On the other hand, it is well known that localization phenomena occur also in periodic nonlinear media at both classical [12], and quantum levels [13]. A fascinating example in optics is the gap or self-transparency solitons arising from mutual compensation of Bragg grating dispersion and Kerr nonlinearities. Evidence of the formation of a slow gap soliton has been recently reported in a glass fiber filter [14].

It stems from the well established notion of quadratic solitons and Bragg localization to investigate the existence of parametric (i.e., quadratic) gap solitons in periodic media with $\chi^{(2)}$ nonlinearities (see, e.g., Refs. [15–17]). The problem appears rather challenging due to the large number of involved wave envelopes: four in the simplest case of SHG. Nevertheless, SHG in a doubly resonant Bragg grating allows for the formation of two-color gap solitons which are purely parametric in nature (not relying on any equivalent cubic nonlinearity) [18,19]. These soliton envelopes are reminiscent of parametric solitons in uniform media with second-order dispersion (i.e., they obey equations of identical form). However, this perturbative result is valid only when both carriers are in the vicinity of the edges of the linear stop gaps associated with the Bragg resonances. Conversely, in order to assess the experimental feasibility of parametric gap solitons, it is important to describe analytically localized solutions which span the entire gaps. Those solutions reported to date are not of true parametric origin in the sense that they require the propagation effects at SH to be negligible, causing SHG to give rise to cubic equivalent terms [17,20]. In Bragg gratings this implies a proper engineering in order to induce a strong Bragg resonance at SH [20]. In this paper we show that more general stationary solutions can be explicitly obtained. They are experimentally accessible being robust under the action of perturbations, and valid also in the particular case of vanishing Bragg coupling at SH, namely in singly resonant gratings for which the technology is well developed [12,15].

Let us consider a two-color field $E(z,t)=\Sigma_{m=1,2}E_m^+(z,t)\exp(ik_mz-im\omega_0t)+E_m^-(z,t)\exp(-ik_mz-im\omega_0t)$ with fundamental (FF, ω_0), and SH ($2\omega_0$) carrier frequencies. Here the $+$ $(-)$ indicates forward (backward) propagation in a material with $\chi^{(2)}$ nonlinearity and z-periodic variation of the linear susceptibility with period $\Lambda=\pi/\beta_0$. In the rotating-wave and quasiscalar approximations the coupled-mode equations which govern the envelope propagation can be casted in the dimensionless form [18]

$$\begin{aligned} -i\phi_{1\tau}^{\pm}&=[\delta H/\delta(\phi_1^{\pm})^*]=\pm iv_1\phi_{1\xi}^{\pm}+\Delta_1\phi_1^{\pm}\\ &\quad+v_1\kappa_1\phi_1^{\mp}+\phi_2^{\pm}(\phi_1^{\pm})^*,\\ -i\phi_{2\tau}^{\pm}&=[\delta H/\delta(\phi_2^{\pm})^*]=\pm iv_2\phi_{2\xi}^{\pm}+\Delta_2\phi_2^{\pm}\\ &\quad+v_2\kappa_2\phi_2^{\mp}+[(\phi_1^{\pm})^2/2], \end{aligned} \tag{1}$$

where $\xi\equiv\Gamma_1z$ and $\tau\equiv\Gamma_1V_1t$ are normalized distance and time, respectively, Γ_m and $V_m^{-1}=\partial k/\partial\omega|_{m\omega_0}$ $(m=1,2)$ being the grating coupling strengths and the inverse group velocities. The envelope amplitudes are $\phi_m^{\pm}=c_mE_m^{\pm}\exp[\pm i(k_m-m\beta_0)z]$, where $c_1=\sqrt{2V_2\chi_1\chi_2/(V_1\Gamma_1^2)}$, $c_2=\chi_1/\Gamma_1$, $|E_m|^2$ gives the intensity and $\chi_m[W^{-1/2}]$ are usual nonlinear coefficients [18]. The field evolution depends on four normalized parameters: the SH to FF ratios of coupling strengths $\kappa_2=\Gamma_2/\Gamma_1$, and group velocity $v_2=V_2/V_1$, and FF and SH detunings from Bragg resonances $\Delta_m\equiv V_m(k_m-m\beta_0)/(V_1\Gamma_1)$. We also introduce $v_1=\kappa_1=1$ to simplify the notation. The SHG wave vector mismatch, which does not explicitly appear in Eqs. (1), is given in terms of the detunings $\Delta_{1,2}$ as $\Delta k\equiv k_2-2k_1=\Gamma_1[(V_1\Delta_2/V_2)-2\Delta_1]$. Notice that, although Eqs. (1) describe a doubly resonant Bragg grating (or an anharmonic grating, the SH being resonant with the grating SH), in the important case of a single Bragg resonance at FF (i.e., nearly harmonic grating, $\Gamma_2=0$) they hold valid with $\kappa_2\to0$, the other definitions remaining unchanged.

Since Eqs. (1) are invariant with respect to temporal (τ) or space (ξ) translations, and phase scaling $\phi_m^{\pm}\to\phi_m^{\pm}\exp(im\psi_0)$, they conserve the Hamiltonian $H=\int_{-\infty}^{+\infty}\mathcal{H}d\xi$ where

$$2\mathcal{H}=\sum_{m=1,2} i v_m[\phi_m^-(\phi_{m\xi}^-)^*-\phi_m^+(\phi_{m\xi}^+)^*]$$
$$+2\Delta_m(|\phi_m^+|^2+|\phi_m^-|^2)+2v_m\kappa_m\phi_m^-(\phi_m^+)^*$$
$$+(\phi_2^+)^*(\phi_1^+)^2+(\phi_2^-)^*(\phi_1^-)^2+\text{c.c.}$$

as well as the energy (or mass) $N=\int_{-\infty}^{+\infty}\Sigma_{m=1,2}m(|\phi_m^+|^2+|\phi_m^-|^2)d\xi$ and the momentum

$$M=i\int_{-\infty}^{+\infty}\left[\sum_{m=1,2}\phi_m^+(\phi_{m\xi}^+)^*+\phi_m^-(\phi_{m\xi}^-)^*-\text{c.c.}\right]d\xi.$$

Furthermore, the energy and its flux are related by the equation

$$\partial_\tau(|\phi_1^+|^2+|\phi_1^-|^2+2|\phi_2^+|^2+2|\phi_2^-|^2)$$
$$=-\partial_\xi(v_1|\phi_1^+|^2-v_1|\phi_1^-|^2+2v_2|\phi_2^+|^2-2v_2|\phi_2^-|^2).$$

We seek standing-wave localized solutions of Eqs. (1),

$$\phi_m^\pm(\xi,\tau)=\sqrt{v_1v_{3-m}}u_m^\pm(\xi)\exp(im\lambda\tau),\quad m=1,2;\qquad(2)$$

which is equivalent to solve the variational problem $\delta(H+\lambda N)=0$. The envelopes $u_{1,2}^\pm$ obey the following system of ordinary differential equations (ODEs)

$$\mp iu_{1\xi}^\pm=[\partial H_0/\partial(u_1^\pm)^*]=\delta_1u_1^\pm+u_1^\mp+(u_1^\pm)^*u_2^\pm,\qquad(3)$$

$$\mp iu_{2\xi}^\pm=[\partial H_0/\partial(u_2^\pm)^*]=\delta_2u_2^\pm+\kappa_2u_2^\mp+[(u_1^\pm)^2/2],\qquad(4)$$

with $\delta_m\equiv(\Delta_m-m\lambda)/v_m$. At low power, Eqs. (3) and (4) decouple and the linear solutions $u_m^\pm$ at frequency $m(\omega_0-\lambda)$ shows the existence of two distinct forbidden gaps (where the waves are exponentially decaying inside the grating) for $|\delta_m|<\kappa_m$, with $m=1,2$ [18]. Equations (3) and (4) preserve the translational and phase invariance and conserve the Hamiltonian

$$H_0=\sum_{j=1,2}\frac{\delta_j}{2}(|u_j^+|^2+|u_j^-|^2)+u_1^-u_1^{+*}$$
$$+\kappa_2(u_2^-u_2^{+*})+\tfrac{1}{2}[(u_1^+)^2(u_2^+)^*+(u_1^-)^2(u_2^-)^*]+\text{c.c.}$$

and the photon flux $Q_0=|u_1^+|^2-|u_1^-|^2+2|u_2^+|^2-2|u_2^-|^2$. Since stationary solutions have vanishing flux, it is reasonable to look for spatially localized solutions of Eqs. (3) and (4) in the form $u_{1,2}^+=c_{1,2}^+x_{1,2}(\xi)$ and $u_{1,2}^-=c_{1,2}^-x_{1,2}^*(\xi)$, with $c_{1,2}^\pm=|c_{1,2}^\pm|\exp(i\psi_{1,2}^\pm)$ complex constants. This allows one to reduce Eqs. (3) and (4) to two ODEs, and since compatibility requires $|c_1^+|^2=|c_1^-|^2$ and $|c_1^+|^2=|c_1^-|^2$, we take $|c_{1,2}^\pm|=1$ without loss of generality. From Eqs. (3) and (4), we obtain the system (the dot stands for $\partial/\partial\xi$)

$$i\dot{x}_1+\delta_1x_1+c_{l1}x_1^*+c_{nl}x_2x_1^*=0,$$
$$i\dot{x}_2+\delta_2x_2+\kappa_2c_{l2}x_2^*+c_{nl}^*x^2/2=0,\qquad(5)$$

where $c_{l1,l2,nl}\equiv\exp(i\psi_{l1,l2,nl})$ with $\psi_{l1,l2}\equiv\psi_{1,2}^--\psi_{1,2}^+$ and $\psi_{nl}=\psi_{l1}-\psi_{l2}/2$. The values of the coefficients in Eqs. (5) depend on the choice of the two phases $\psi_{l1,l2}$ which also fix ψ_{nl}. We single out two cases which yield real values for both the linear and the nonlinear terms: (a) $c_{l1}=c_{nl}=c_{nl}^*=-1$, $c_{l2}=1$ (i.e., $\psi_{l1}=\psi_{nl}=\pi$, $\psi_{l2}=0$); (b) $c_{l1}=c_{l2}=c_{nl}=c_{nl}^*=1$ (i.e., $\psi_{l1}=\psi_{l2}=\psi_{nl}=0$). In the following we deal with case (a): all the results can be extended to case (b) through the substitutions $\delta_1,\delta_2,\kappa_2,\xi\to-\delta_1,-\delta_2,-\kappa_2,-\xi$. It is useful to write Eqs. (5) as a canonical Hamiltonian system with two degrees of freedom, by posing $x_{1,2}\equiv x_{1r,2r}+ix_{1i,2i}$ and separating the equations in real and immaginary parts. We obtain

$$\dot{x}_{1r}=-(\partial h/\partial x_{1i})=-(\delta_1+1)x_{1i}+x_{1r}x_{2i}-x_{2r}x_{1i},\qquad(6)$$

$$\dot{x}_{1i}=+(\partial h/\partial x_{1r})=(\delta_1-1)x_{1r}-x_{1r}x_{2r}-x_{1i}x_{2i},\qquad(7)$$

$$\dot{x}_{2r}=-(\partial h/\partial x_{2i})=(\kappa_2-\delta_2)x_{2i}+x_{1r}x_{1i},\qquad(8)$$

$$\dot{x}_{2i}=+(\partial h/\partial x_{2r})=(\kappa_2+\delta_2)x_{2r}-\tfrac{1}{2}(x_{1r}^2-x_{1i}^2),\qquad(9)$$

where the Hamiltonian

$$h=-x_{1r}x_{1i}x_{2i}-\frac{x_{2r}}{2}(x_{1r}^2-x_{1i}^2)+\frac{(\delta_1-1)}{2}x_{1r}^2$$
$$+\frac{(\delta_1+1)}{2}x_{1i}^2+\frac{(\delta_2+\kappa_2)}{2}x_{2r}^2+\frac{(\delta_2-\kappa_2)}{2}x_{2i}^2.$$

The ODEs [Eqs. (6)–(9)] need an additional conserved quantity to be integrable. Although its existence is not supported by physical arguments in the general case, the following relation can be easily verified:

$$(d/d\xi)p=2x_{2i}[(\kappa_2-\delta_2)+2\kappa_2x_{2r}],\qquad(10)$$

with $p\equiv x_{2r}^2+x_{2i}^2+(x_{1r}^2+x_{1i}^2)/2+2x_{2r}$. We therefore distinguish two integrable cases of interest: (c1) $x_{2i}=0$; (c2) $\kappa_2=\delta_2=0$, in which p represents the additional conserved quantity. Note that, in both cases [(c1) and (c2)] we solved the quadratic system [Eqs. (6)–(9)] retaining the ξ derivatives of the SH field. In other words our solutions are such that the SH *is not adiabatically following* the FF. Let us consider first the case (c1) $x_{2i}=0$, corresponding to a real SH envelope x_2. In this case, Eq. (9) yields $x_{2r}=(x_{1r}^2-x_{1i}^2)/[2(\delta_2+\kappa_2)]$ which, to be compatible with Eqs. (6)–(8), requires in turn the *resonance* condition

$$2\delta_1+\delta_2+\kappa_2=0,\qquad(11)$$

which can be satisfied by properly tuning δ_1 and δ_2 for a given value of κ_2, and in particular also for the single Bragg coupling $\kappa_2=0$. By eliminating x_{2r} in Eqs. (6) and (7) and introducing the variables $x_{r,i}\equiv x_{1r,1i}/2\sqrt{|\delta_1|}$ we obtain the self-consistent system

$$\dot{x}_r=-(\partial h_1/\partial x_i);\quad \dot{x}_i=(\partial h_1/\partial x_r),\qquad(12)$$

where the one degree of freedom Hamiltonian is

$$h_1=(\delta_1-1)x_r^2/2+(1+\delta_1)x_i^2/2+(\gamma/4)(x_r^4+x_i^4-2x_r^2x_i^2),$$

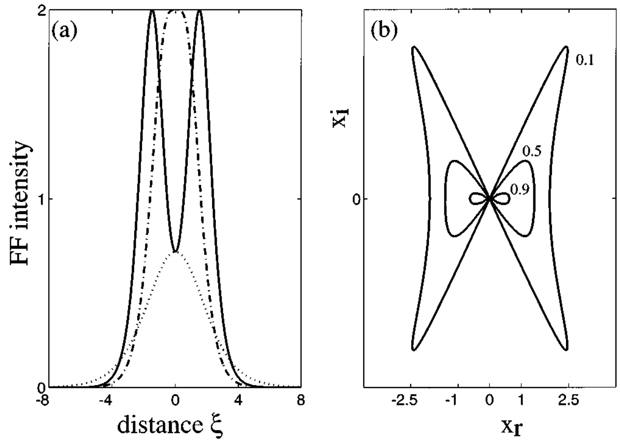

FIG. 1. (a) Soliton FF intensity profiles for $\delta_1=0.9$ (dots); $\delta_1=0.5$ (dot-dashes); $\delta_1=0.1$ (solid), (b) corresponding separatrices in the phase plane (x_r, x_i).

and $\gamma\equiv\mathrm{sign}(\delta_1)$. System (12) depends only on δ_1, and the invariance $\delta_1\rightarrow-\delta_1, x_{r,i}\rightarrow x_{i,r}$ permits one to restrict to values $\delta_1>0$ $(\gamma=1)$. The origin $(x_r,x_i)=(0,0)$ is the only fixed point in Eqs. (12). Therefore, the solitary solutions are expected to be bright-bright waves ($x_{1r}=x_{1i}=x_{2r}=0$ at infinity), which correspond to the invariant $p=0$. To construct the localized solutions explicitly, it is more convenient to exploit the conservation of p and the solution of Eq. (9) to express x_{1r} and x_{1i} in terms of x_{2r} as

$$x_{1r}^2=-x_{2r}^2-2(1+\delta_1)x_{2r}+p;$$
$$x_{1i}^2=-x_{2r}^2-2(1-\delta_1)x_{2r}+p. \qquad (13)$$

It follows from Eq. (8) a decoupled equation for the equivalent particle coordinate x_{2r}, in the potential form $\ddot{x}_{2r}=-\partial V/\partial x_{2r}$, where $V=V(x_{2r})$ is the quartic well

$$V=-\tfrac{1}{2}x_{2r}^4-2x_{2r}^3-[2(1-\delta_1^2)-p]x_{2r}^2+2px_{2r}. \qquad (14)$$

For $p=0$, this equivalent particle model yields the periodic localized solution (so-called snoidal wave)

$$x_{2r}(\xi)=\frac{x_3-x_4A\,\mathrm{sn}^2(\alpha\xi|k)}{1-A\,\mathrm{sn}^2(\alpha\xi|k)}, \qquad (15)$$

where $\alpha=2[(x_1-x_3)(x_2-x_4)]^{-1/2}$, $A=(x_2-x_3)/(x_2-x_4)$, and $x_1>x_2>x_3>x_4$ are the roots of the polynomial $2[E-V(x)]=(x-x_1)(x-x_2)(x-x_3)(x-x_4)$, E being a constant with the meaning of particle total energy. The period of the solution (15) is $\xi_p=4K(k)/\alpha$ with $k^2=A(x_1-x_4)/(x_1-x_3)$. In the limit $x_1=x_2=0$, $x_3=2(\delta_1-1)$, $x_4=-2(\delta_1+1)$, then $\xi_p\rightarrow\infty$. Hence Eq. (15) yields the standing solitary wave

$$x_{2r}(\xi)=\frac{2(\delta_1^2-1)}{1+\delta_1\cosh(2\sqrt{1-\delta_1^2}\xi)}, \qquad (16)$$

which exists for $\delta_1^2<1$, i.e., inside the stop-gap associated with the Bragg resonance at FF. From Eq. (16) and Eqs. (13), we obtain the corresponding FF field intensity

$$|x_1(\xi)|^2=8\delta_1(1-\delta_1^2)\frac{\delta_1+\cosh(2\sqrt{1-\delta_1^2}\xi)}{[1+\delta_1\cosh(2\sqrt{1-\delta_1^2}\xi)]^2}$$

and phase

$$\arg(x_1)=\tan^{-1}[\sqrt{(1-\delta_1)/(1+\delta_1)}\tanh(\sqrt{1-\delta_1^2}\xi)].$$

These FF intensity profiles are shown in Fig. 1(a): for $0.5<|\delta_1|<1$ they exhibit a single hump. As the gap edges $|\delta_1|=1$ are approached, the peak intensity $8\delta_1(1-\delta_1^2)/(1+\delta_1)$ decreases and the width $(1-\delta_1^2)^{-1/2}$ increases. For $|\delta_1|<0.5$ the solitary envelopes are double-humped, corresponding to separatrices of Eqs. (12) with characteristic butterfly shapes [see Fig. 1(b)].

Let us now consider the standing solitary wave solutions in the other integrable limit (c2), namely for $\delta_2=\kappa_2=0$. In this case the grating shows a negligible Bragg coupling at SH and satisfies the resonance condition $\Delta k+\Delta\beta=0$ (or $k_2-k_1=\beta_0$), that can be easily fulfilled at phase-matching and perfect Bragg resonance. Using Eqs. (10), we eliminate the variable x_{2r} and obtain the following self-consistent system for $x_{1r,1i}$

$$\ddot{x}_{1r}=-(\partial U/\partial x_{1r}); \quad \ddot{x}_{1i}=-(\partial U/\partial x_{1i}), \qquad (17)$$

where $U=(x_{1r}^4+x_{1i}^4+2x_{1r}^2x_{1i}^2)/4-q^2(x_{1r}^2+x_{1i}^2)/2$ is a potential well, and $q^2\equiv1-\delta_1^2+p$. Equations (17) are formally identical to the ODEs from the focusing Manakov system of coupled nonlinear Schrödinger equations [21], which describes the evolution of a two component envelope soliton (for instance in fibers with random birefringence). The solitary solution of Eqs. (17) is $x_1=x_{1r}+ix_{1i}=\sqrt{2}q\,\mathrm{sech}(q\xi)\exp(i\psi_0)$, where ψ_0 is an arbitrary phase. The SH component can be easily obtained by solving the coupled equations

$$\dot{x}_{2r}=x_{1i}x_{1r}; \quad \dot{x}_{2i}=\tfrac{1}{2}(x_{1i}^2-x_{1r}^2), \qquad (18)$$

and it turns out to be a dark envelope $x_2=x_{2r}+ix_{2i}=-i\tanh(q\xi)\exp(i2\psi_0)$. We conclude that in this case the two-

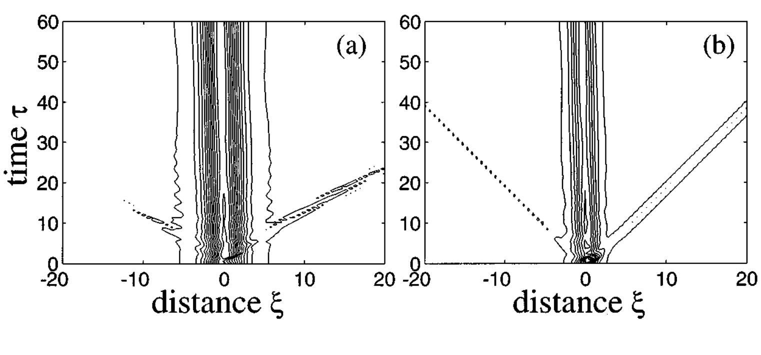

FIG. 2. Spatiotemporal evolution of the FF (a) and SH (b) intensity from a spatially perturbed localized soliton, on perfect resonance $\Theta(\lambda=0)=0$. Here $\kappa_2=0$ (single Bragg resonance), $\delta_1=\Delta_1=0.8$, $\delta_2=\Delta_2/0.5=-2\delta_1=-1.6$.

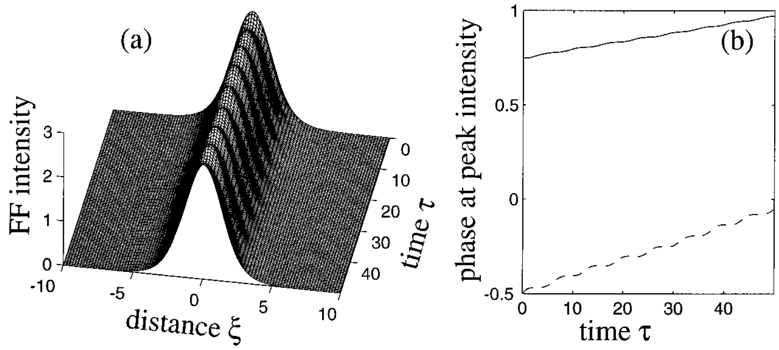

FIG. 3. Temporal evolution for a soliton with a deviation of 20% from resonance, i.e., $\Theta(\lambda=0)=0.2$: (a) FF intensity; (b) FF (solid) and SH (dashed) phase at peak intensity. Here $\delta_1=\Delta_1=0.8$, $\kappa_2=0.5$, $v_2=V_2/V_1\sim\omega_0/2\omega_0=0.5$.

color soliton is a bright-dark pair, thereby requiring strong SH excitation at both grating boundaries.

Finally, let us address the important problem of stability of the bright gap solitons (16). Since a general stability criterium has not even been developed in the simpler case of gap solitons of Kerr materials (e.g., fibers [12]), we resort to numerically integrate Eqs. (1), studying the temporal evolution of the localized profiles in the presence of a perturbation. To this extent we introduce the quantity

$$\Theta(\lambda)\equiv 2\delta_1+\delta_2+\kappa_2=2\Delta_1+\Delta_2/v_2+\kappa_2-2\lambda[1+(1/v_2)],$$

which quantify the deviation from the resonance condition of the case (c1): for any given frequency shift λ, the resonance condition (11) is fulfilled for $\Theta(\lambda)=0$. We performed two distinct sets of numerical experiments: in the first we chose $\Delta_{1,2}$, κ_2, and v_2 to satisfy the resonance $\Theta(\lambda=0)=0$. In this case, if the initial profile matches exactly the bound solution (16), we observe time-independent evolutions. We assessed the robustness of gap solitons by adding a spatial perturbation (we considered both odd and even perturbations and a wide range of parameter values). A typical result is shown in Fig. 2 for a single Bragg resonance at FF ($\kappa_2=0$) and an even perturbation with Gaussian shape and about 30% of the soliton energy component at FF. After a transient during which energy is emitted in the form of radiation (linear waves), the stationary wave pair induces self-transparency and remains trapped in stable fashion. In Fig. 2 the emission of linear waves is stronger at SH because it experiences no linear Bragg effect, and the low-power propagation at SH inside the grating is allowed. In the case of one-hump solitons, i.e., for $0.5<|\delta_1|<1$, the behavior shown in Fig. 2 persists for even stronger perturbations (up to 50% of the soliton energy). Conversely, the solitary waves with double peak at FF ($0<|\delta_1|<0.5$) are more sensitive and easily destroyed by relatively strong perturbations.

In the second set of numerical tests we chose the parameters so that $\Theta(\lambda=0)\neq 0$, using an initial profile matching the solitary solution (16) for the chosen value δ_1. In this case we expect the robustness to induce the soliton acquiring a frequency shift $\lambda\neq 0$ such that $\Theta(\lambda\neq 0)=0$. For single-humped solitons ($0.5<|\delta_1|<1$) this is confirmed by our numerical results (see the example in Fig. 3). As shown, the spatial profile of the soliton exhibits weak oscillations around an average solution with a temporal slope of the phase due to a $\lambda\neq 0$. Again, the double-humped solutions ($|\delta_1|<0.5$) are more sensitive to the resonance condition, being easily destroyed by deviations Θ of a few percent.

In conclusion we have reported exact solutions describing standing self-transparency or gap parametric solitary waves, which appear to be accessible with the state of the art technology of Bragg gratings in standard $\chi^{(2)}$ materials for SHG [15]. For instance, a soliton trapped over a distance $\xi=\xi_L=\Gamma_1 L\simeq 10$ (a sample $L=2$ cm long with a coupling strength $\Gamma_1=5\ \mathrm{cm}^{-1}$), requires a peak intensity $|E_1|^2=(\Gamma_1/2\chi_1)^2 x_1^2(\xi=0)$ less than 1 GW/cm^2 for typical nonlinear coefficients $\chi_1\sim\chi_2\sim 5\times 10^{-4}$ W$^{-1/2}$ [4,5]. Less intense and wider solitons require stronger (or longer) gratings. Wave guides seem suitable candidates to overcome diffraction effects over the characteristic localization lengths [5,14].

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